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Holographic determination of translation and rotation.

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HOLOGRAPHIC DETERMINATION OF TRANSLATION AND ROTATION

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THESIS

HOLOGRAPHIC DETERMINATION OF TRANSLATION AND ROTATION

by

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June 1970

Holographic Determination
of
Translation and Rotation

by

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ABSTRACT

Analytic investigation was made of the light reflected by translated and rotated objects, and expressions were obtained for the interference patterns produced by this light. The technique of holography was used to experimentally measure fringe patterns produced by this interference, and to experimentally determine translation and rotation.

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I. INTRODUCTION

The process of recording a particular scene on paper for storage or study has remained relatively constant in the hundred or so years since photography was developed. It is common knowledge that in photography, light from the scene falls on some detector which records the intensity of that light. The result of the process is a two-dimensional reproduction of the scene. In 1948, however, Dennis Gabor showed that by mixing the light from the scene with a second light wave, a coded intensity distribution would form on the photographic plate. This distribution could later be reconstructed to give a reproduction of the original object. He called this type of picture a hologram. [Ref. 1]

The holographic process depends to some extent on the availability of a source of coherent light; it is necessary that the light on the scene and the reference beam, mentioned above, be in phase with each other when they strike the plate. Thus the advent of the laser has made holography a more practical optical method, and since 1962, there has been much advancement in this field.

The general method of obtaining a hologram varies from laboratory to laboratory, but basically it is as follows. Attention is called to Fig. 1 for purposes of this explanation. Coherent light from source A is split into two segments at B. Part of the split beam is directed to the object O, and from there to the photographic plate at C. The remainder of the beam proceeds from B directly, or via mirrors, to C. Embellishments may be added to spread, focus, clarify or otherwise alter the light

beam or its path, but the net result is basically the same. One consideration applies to all cases: the difference in path length between 'BOC' and 'BC' must be less than the coherence length of the light involved, or there will be no hologram. Here is the great advantage of, or even necessity for, the laser; no other light source has the same readily available large coherent length.

In the above discussion, the beam which strikes the scene and then passes on to the plate is the object beam; the other is the reference beam. Once the plate has been developed - a process no different than regular photographic developing - the hologram is made permanent. Thereafter, by placing the plate in a position where light from a point source strikes it, at the same angle that the original reference beam struck it, the 'code' is unscrambled and the scene is visible. The important difference between this reproduction of a scene and that of normal photography is that in the hologram the scene retains the three-dimensional characteristics of the original.

If the original scene is a single object it is obvious that it may be translated or rotated relative to its original position, and that this movement may be made controllable to the order of the wavelength of laser light. In this case, one type of hologram, called a "Real Time" hologram, after development can be positioned relative to the object exactly as it was before the exposure. Once this is done, a look through the hologram shows the original object and the reconstructed image in the hologram, superimposed on each other. If at this time the original is moved a small

amount, the light reflecting from it will not be exactly the same as it was during the exposure; hence, the new light reflection pattern and the one in the hologram reconstruction will interfere. This interference will be in part additive and part subtractive, with the net result that a pattern of fringes will be visible. If the movement of the original is changed, the fringe pattern will also change, and thus the relation of the fringes to the amount and type of movement may be studied.

In the same type of experiment, it is possible to obtain a double-exposure hologram in the following manner: in the original formation of the hologram, the photographic plate is double-exposed such as camera film may be double-exposed. That is, two pictures are recorded on the same plate. The first recording is of the object under consideration; the second recording is of the same object after movement through some distance. In this case the interference pattern is caught on the plate and made a permanent part of the hologram. This type of hologram is called "frozen." The interference pattern visible in the frozen hologram is the same as one seen in a "real-time" hologram if the object is moved the same amount, but there are some significant differences between the two types. The "frozen" hologram is limited in that only one movement may be studied at one time; to examine a second movement and its resulting fringe pattern a new hologram must be made. The "real-time" hologram is severely limited in that movements of only one wavelength of laser light can significantly alter the fringe pattern; hence, vibrations are a serious problem.

Work in the field of the effect of movement on the hologram has been done by Haines and Hildebrand at the University of Michigan [Ref. 2], Aleksandrov and Bonch-Bruevich in the U.S.S.R. [Ref. 3], and others. Their papers show mainly the results obtained from holograms of objects undergoing strain. The resulting fringe pattern was examined and, using certain optical principles, the amount of strain was deduced. In this paper an examination of these optical principles will be made, by obtaining a series of frozen holograms and looking at the trends shown, relating the amount and type of displacement the object is subjected to, and the fringe pattern that results from this displacement.

II. ANALYTICAL DEVELOPMENT

In Fig. 2 the general case of a displacement of a point P by a distance d to a point Q is now considered. This development follows that given by J. E. Sollid [Ref. 4].

The phase changes along paths k_1 k_2 and k_3 k_4 are given by

$$\delta_1 = \vec{k}_1 \cdot \vec{r} + \vec{k}_2 \cdot (\vec{R} - \vec{r}_1)$$

$$\delta_2 = \vec{k}_3 \cdot \vec{r}_3 + \vec{k}_4 \cdot (\vec{R} - \vec{r}_3)$$

It is assumed that

$$\vec{k}_3 = \vec{k}_1 + \Delta \vec{k}_1$$

$$\vec{k}_4 = \vec{k}_2 + \Delta \vec{k}_2$$

The phase difference in terms of these variables is the

$$\delta = \delta_1 - \delta_2 = (\vec{k}_1 - \vec{k}_2) \cdot (\vec{r}_1 - \vec{r}_3) - \Delta \vec{k}_1 \cdot \vec{r}_3 - \Delta \vec{k}_2 \cdot (\vec{R} - \vec{r}_3)$$

$$|r_1| \simeq |r_3| >> |d| = |r_3 - r_1|$$

and in this case $\vec{\Delta k}_1$ and $\vec{2k}_2$ are perpendicular to $\vec{r}_3$ and $(\vec{R} - \vec{r}_3)$

$$\therefore \qquad \delta = (\vec{k}_1 - \vec{k}_2) \cdot (\vec{r}_1 - \vec{r}_2)$$

In the case of two dimensions where the incident and scattered light are co-planar with the displacement one has with

$$\vec{d} = \vec{r}_3 - \vec{r}_1$$

$$\delta = -(\vec{k}_1 - k_2) \cdot \vec{d}$$

For the case of simple rotation one has

$$d = y \tan\beta \doteq y\beta$$

where y is measured from the axis of rotation. In terms of the angles θ_1 and θ_2 given in Fig. 1 this equation then yields

$$\delta = k y \beta (\sin\theta_1 + \sin\theta_2)$$

$$k = \frac{2\pi}{\lambda}$$

where λ is the wavelength of the light.

Two points Y and Y' are now sought on the object so that light from each arriving at the observer will have a relative phase difference of 2π. That is, the phase difference of Y minus the phase difference of Y' will be 2π. Thus:

$$\delta - \delta' = 2\pi = \frac{2\pi}{\lambda}\left[\beta\left(\sin\theta_1 + \sin\theta_2\right)\left\{y - y'\right\}\right]$$

$$(y - y') = \frac{\lambda}{\beta}\left(\sin\theta_1 + \sin\theta_2\right)^{-1}$$

$$D = \frac{\lambda}{\beta}\left(\sin\theta_1 + \sin\theta_2\right)^{-1}.$$

This value, D, corresponds to the distance between fringes for various source and observer angles θ_1 and θ_2. This statement is justified as follows: the phase difference of 2π corresponds to maximum additive interference between the two light waves. On the object there exists several points Y which reflect light that arrives at the observer when the wave is at a maximum. The light from the corresponding point Y' will also arrive at the observer at a maximum then, and the interference will produce an even greater intensity. Thus at the observer are several areas of maximum light intensity, separated by regions of lesser intensity; these bands are called fringes, and the distance between the bright

regions is D, as shown above.

Figure 3 shows the case of pure translation, where a point initially at A has been translated to A'. It is assumed that A and A' radiate light omnidirectionally and that the light incident to the two points is a plane wave. It is also assumed that the light arriving at A is in phase with that arriving at A'. Furthermore, the translation is parallel to the plane of the observer, so that AA' is parallel to P_oP. Midway between A and A' a line is constructed perpendicular to AA', of length R, to P_o. Light is considered to be reflected from A and A' to some point P, in the plane of the observer, at distance Y from P_o. The path difference of light from A to P and light from A' to P is:

$$A'P - AP$$

line AS is constructed so that AP and SP are equal. Thus:

$$R \gg\gg X$$

$$ASA' \sim 90^\circ$$

$$\phi \sim \theta$$

$$A'P - AP = X \sin \phi$$

$$A'P - AP = X \sin \theta$$

$$\theta \sim Y/R$$

$$A'P - AP = X \frac{Y}{R}$$

The phase difference between light reflected from A and A', when viewed at P is:

$$\delta = \frac{2\pi}{\lambda}\left(x \frac{y}{R}\right)$$

Seeking once again two points P and P' where light reflected from A and A' has a relative phase difference of 2π, the distance between the two points will represent the fringe separation. This is given by:

$$2\pi = \delta' = \frac{2\pi}{\lambda}\left[\frac{x}{R}\left(y - y'\right)\right]$$

$$y - y' = \frac{\lambda R}{x}$$

$$D = \frac{\lambda R}{x}$$

III. EXPERIMENTAL PROCEDURE

Using the expressions developed in the previous section, plots were made of expected fringe separation versus various rotations, in one case, and translations in the other. A series of experiments was then executed, designed to measure rotation or translation and the resultant fringe separation. The results of these experiments were graphed on the theoretical curves.

Figure 4 shows the arrangement of apparatus for the experiments. As shown, the source angle, θ_1, was 30 degrees; θ_2 was 90 degrees, and the distance from the object to the plate, R, was 6-$^{11}/_{32}$ inches. The reference beam approached the plate from an angle of 45 degrees; experience showed that this angle gives the greatest ease in reconstruction. The light source was a Helium-Neon gas laser, with wavelength 6328 Angstroms. The object was a high precision mirror mount manufactured by the Oriel optics corporation; the mount had the capability of rotation about two axes and translation in two directions. The photographic plates used were manufactured by the Agfa-Scienta Company, and were of high precision quality, especially designed for holography.

For examination of the results of rotation, the object was rotatable, as it was placed on the optical bench used, about the vertical axis and about the axis parallel to the plate. Rotation about the vertical axis was used, and was measured by means of a micrometer dial on the object, readable to 10 arc-seconds. During the course of the experiment, a plate was first exposed to light reflecting from the object for an 8 second

period. This process was repeated for several different rotations.

Experimentation for the translation case was accomplished in a similar manner. Translation was made parallel to the photographic plate, with measurability to .01 millineter.

To examine the fringe patterns produced in this translation or rotation, an interesting characteristic of holograms was utilized. The simplest method to examine a hologram is to reposition it in the experimental arrangement and allow the reference beam to strike it, insuring that the original reference angle is used. This will illuminate the entire plate and, if the hologram were properly taken, will reconstruct the original scene over the entire plate. In the reconstruction the object takes on its original three-dimensional character, and the fringes are visible as dark lines infront of the object. In this arrangement, however, it is difficult to accurately measure the fringe separation.

It is a characteristic of the hologram that the complete scene is reconstructed in each part of the hologram; in order words, if the hologram were accidentally broken into a dozen pieces, each piece would reconstruct the entire scene exactly like the original hologram. If a thin beam of light were used as a reference beam to reconstruct the hologram, another characteristic appears: the small part of the hologram touched by the narrow beams not only reconstructs the entire scene, it acts as a lens. The scene is magnified so that larger-than-life photographs may be made. A ground glass plate placed behind a hologram reconstructed in this manner may be moved and positioned so that the desired amount of

enlargement will appear.

In making these enlargements another feature is exposed. In holograms taken of rotation, as the scene was enlarged, the distance between fringes also grew. This demonstrates the fact that these fringes are located very near the surface of the object in space. For translation holograms, however, the effect is different: no matter how large the scene is made, the distance between the fringes remains the same. This shows that these fringes are localized at infinity. Further investigation of this phenomenon can be made by moving the hologram from side to side. Fringes formed by rotation will move with the object, but those formed by translation will remain fixed, while the object moves behind them.

Figure 5 is a schematic of the arrangement used to reduce the holograms to a form where the fringe separation could be accurately measured. The camera used was placed so that the scene would be expanded to life size; this was necessary so that the picures would show accurately the fringe separation for the rotational case.

There were several possible sources of error in the experimental process. While the amount of rotation during the experiment was measured fairly accurately, the fact that the micrometer was measurable to only 10 arc-seconds, and that most of the rotation was accomplished in near darkness, led to measured rotations accurate to only plus-or-minus 1 arc-second. Another error may have been introduced in the latter phase of the experiment, that of measuring the fringe separation. Accuracy of the measuring device was good, but to make the measurement

it was necessary to guess at the location of the center of the fringes. Even averaging several measurements could not alleviate the total effect of this guessing, and therefore an error of 4% was applied to fringe measurement. For the translation case, the value of R was accurate only to plus-or-minus 1/32 inch.

IV. PRESENTATION OF DATA

Results of the experimentation were tabulated and graphed and are shown on following pages. Table 1 gives calculated fringe separation for various rotations and shows, where appropriate, the experimental result obtained for that rotation. Error figures are included in the experimental column. Table 2 shows the same data for translation, while Figs. 6 and 7 are graphical displays of this data.

Some of the photographs obtained are also shown in Appendix B. Examination of them gives some feeling for the difficulty of accurately determining the exact center of the fringe. These pictures also give some idea of the actual appearance of the fringes in the hologram.

TABLE I

THEORETICAL AND EXPERIMENTAL FRINGE DISPLACEMENT FOR ROTATION

SOURCE ANGLE = 30 DEGREES
VIEWING ANGLE = 90 DEGREES

ROTATION, β ARC-SECONDS	THEORETICAL D INCHES	EXPERIMENTAL D ERROR INCLUDED
1.00	3.4259	
3.00	1.1420	
3.54	.9720	.9890 ± .0495
4.00	.8565	
5.00	.6852	
5.42	.6350	.6450 ± .0320
7.87	.4378	.4500 ± .0225
10.00	.3426	
11.80	.2918	.2950 ± .0150
15.00	.2284	
15.75	.2185	.2205 ± .0110
19.70	.1745	.1750 ± .0087
20.00	.1713	
23.60	.1458	.1420 ± .0071
25.00	.1370	
27.60	.1246	.1250 ± .0061
30.00	.1142	
40.00	.0856	
50.00	.0685	
60.00	.0571	

TABLE II

THEORETICAL AND EXPERIMENTAL
FRINGE DISPLACEMENT FOR PARALLEL TRANSLATION

R = 6.34375

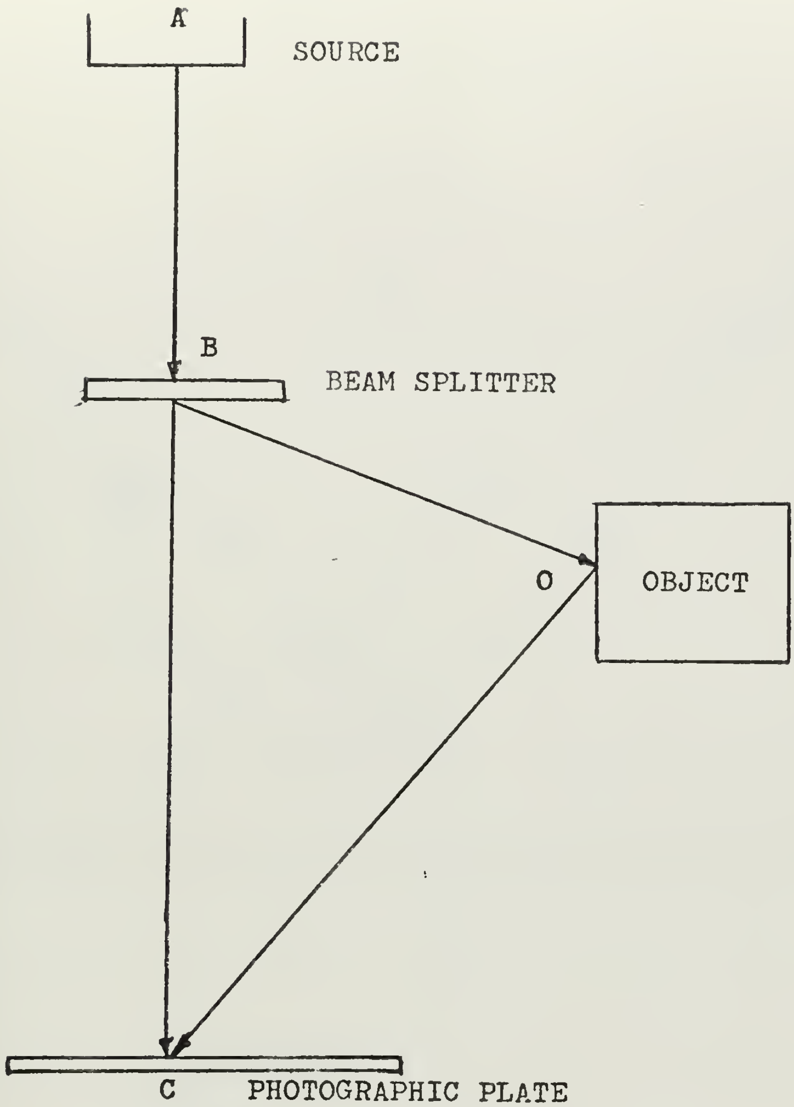
TRANSLATION, X MILLIMETERS	THEORETICAL D INCHES	EXPERIMENTAL D ERROR INCLUDED
.01	.406	
.02	.203	.210 ± .008
.03	.135	.136 ± .005
.04	.101	
.05	.0812	.081 ± .003
.06	.0675	
.07	.0580	
.08	.0507	.050 ± .002
.09	.0451	
.10	.0406	.042 ± .0016
.11	.0369	
.12	.0338	
.13	.0312	
.14	.0290	
.15	.0271	.027 ± .001

V. CONCLUSIONS

The favorable comparisons between expected and experimental results, as shown in the previous section, leads to the conclusion that small deformations can indeed be measured accurately using the holographic technique. With this in mind several further conclusions may be drawn. One is that in laboratory experiments where a small deformation is present but difficult to measure by conventional methods, the hologram can be used to measure the deformation. With this tool deformations of the order of several hundred microns can be accurately measured. Use of stroboscopic lighting and other tools could aid in measureing flutter and other vibrations. Simple static comparisons of manufactured copies to a hologram of the original could greatly aid in quality control in an assembly process.

In a broader sense, it has been shown that the hologram is a practical tool, as useful in the engineering world as it has been in the optical realm.

FIGURE 1



SAMPLE SCHEMATIC
FOR A HOLOGRAM

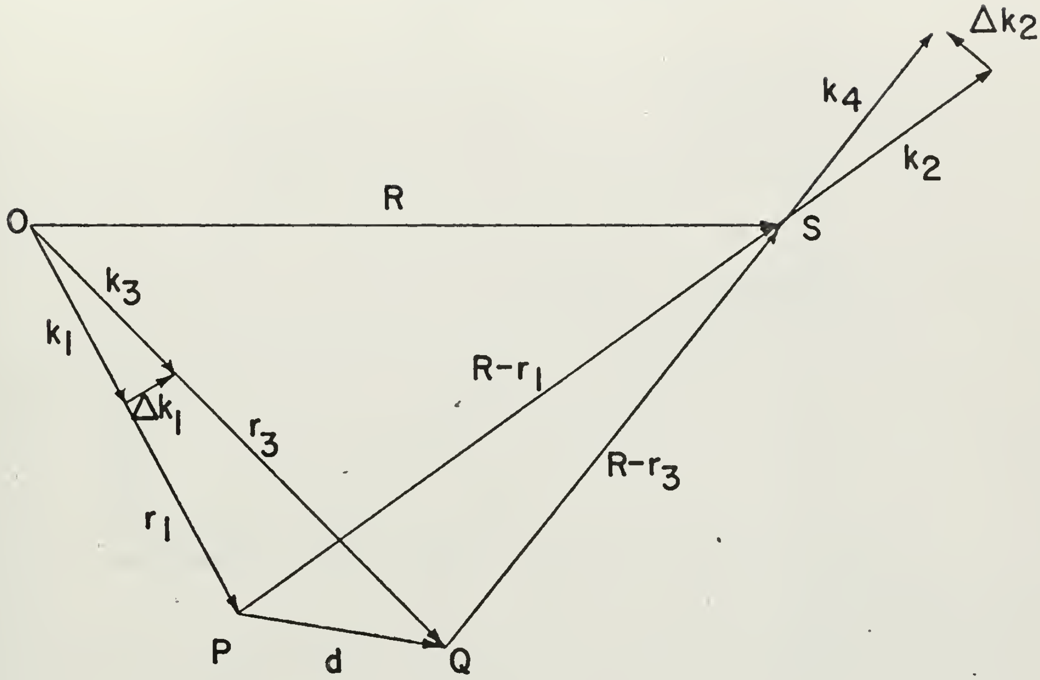
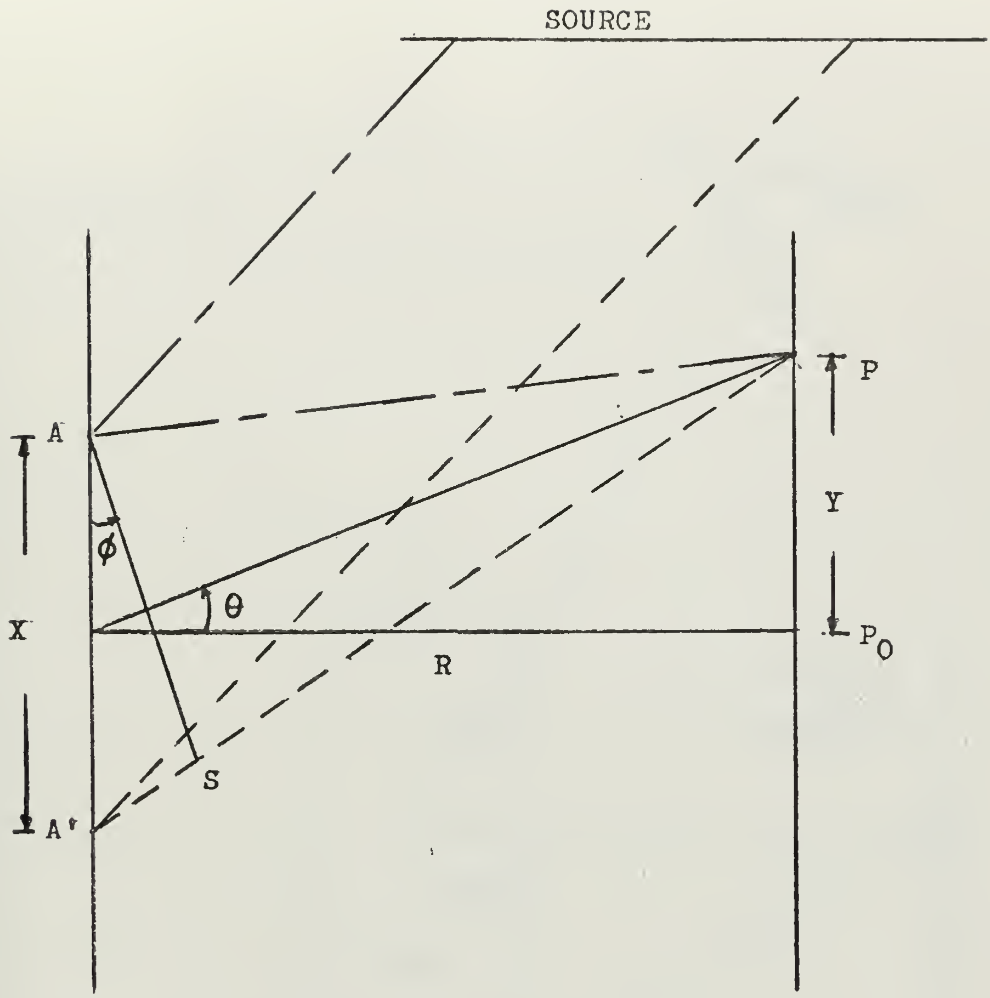


FIGURE 2

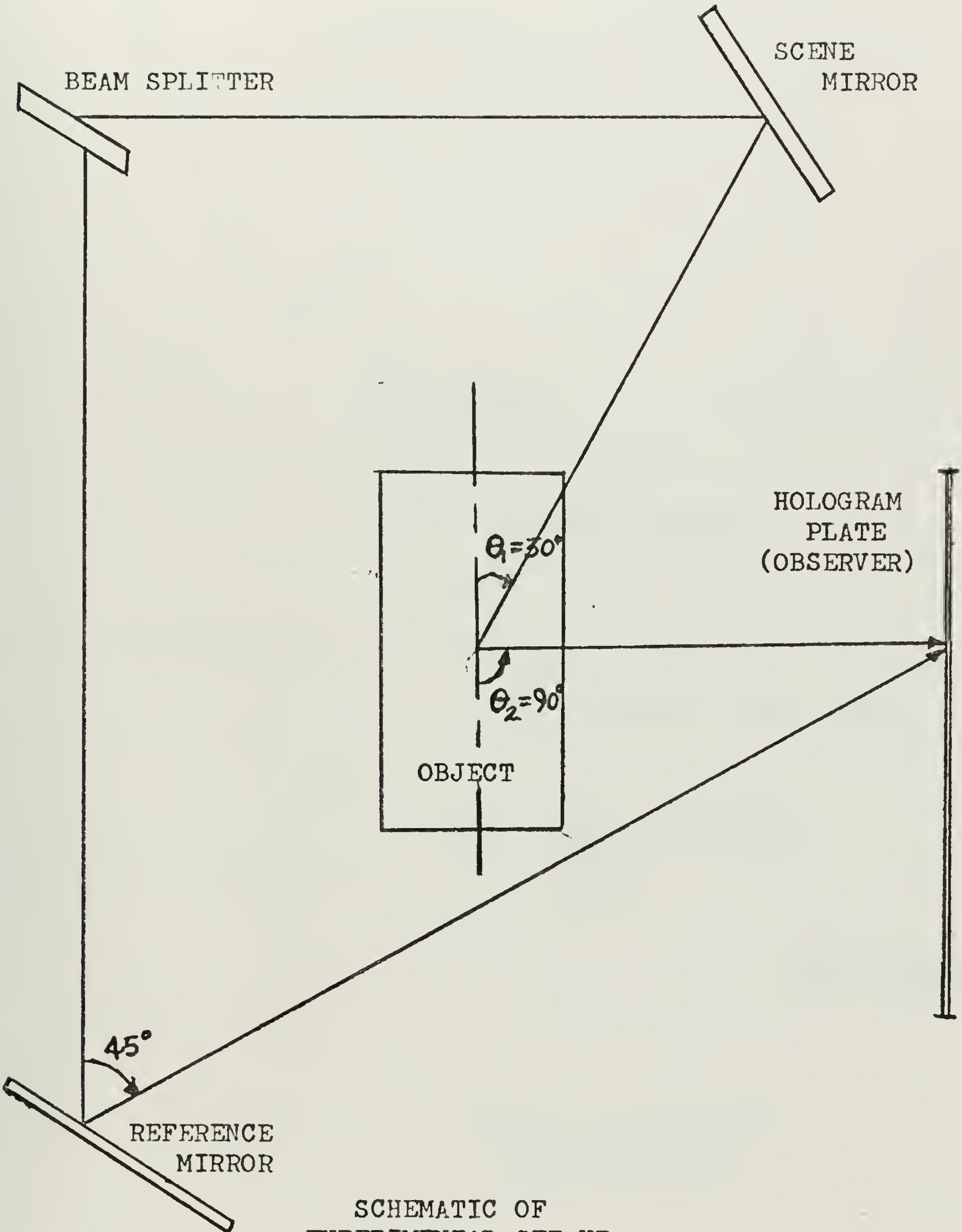
GENERAL DISPLACEMENT

FIGURE 3



LIGHT PATHS
FOR TRANSLATION

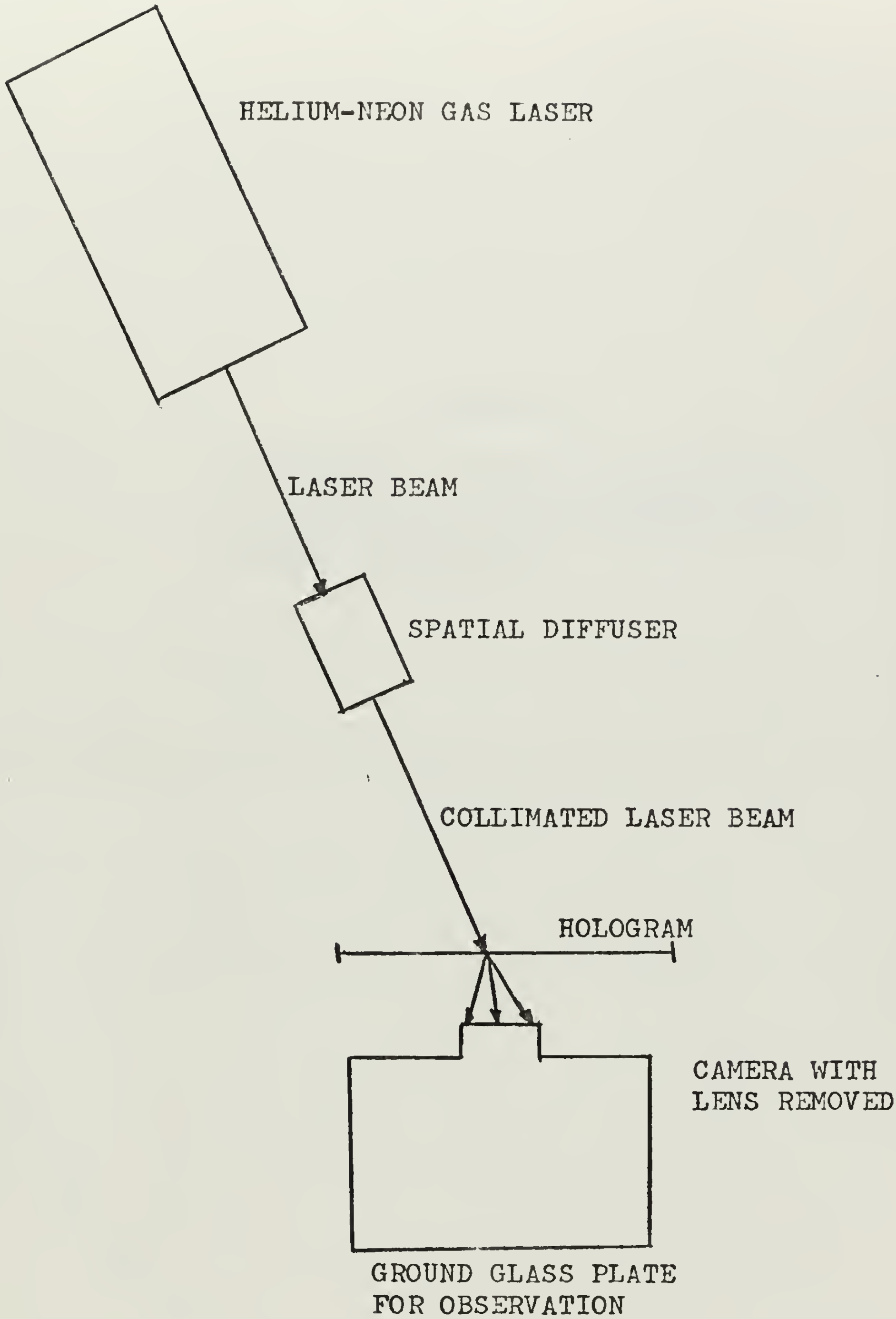
FIGURE 4



SCHEMATIC OF
EXPERIMENTAL SET-UP

FIGURE 5

ARRANGEMENT FOR
REDUCTION OF HOLOGRAMS



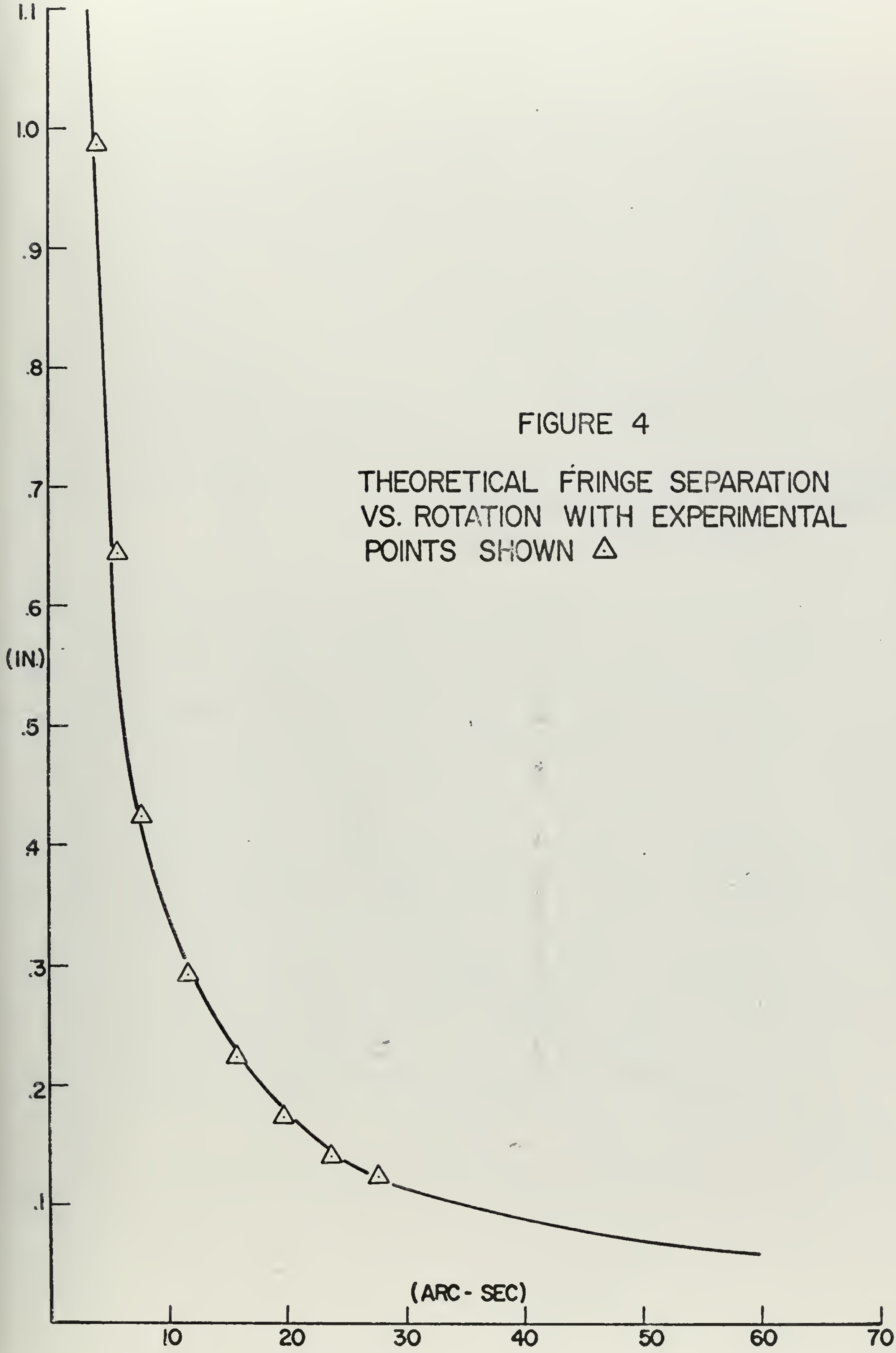


FIGURE 4

THEORETICAL FRINGE SEPARATION VS. ROTATION WITH EXPERIMENTAL POINTS SHOWN △

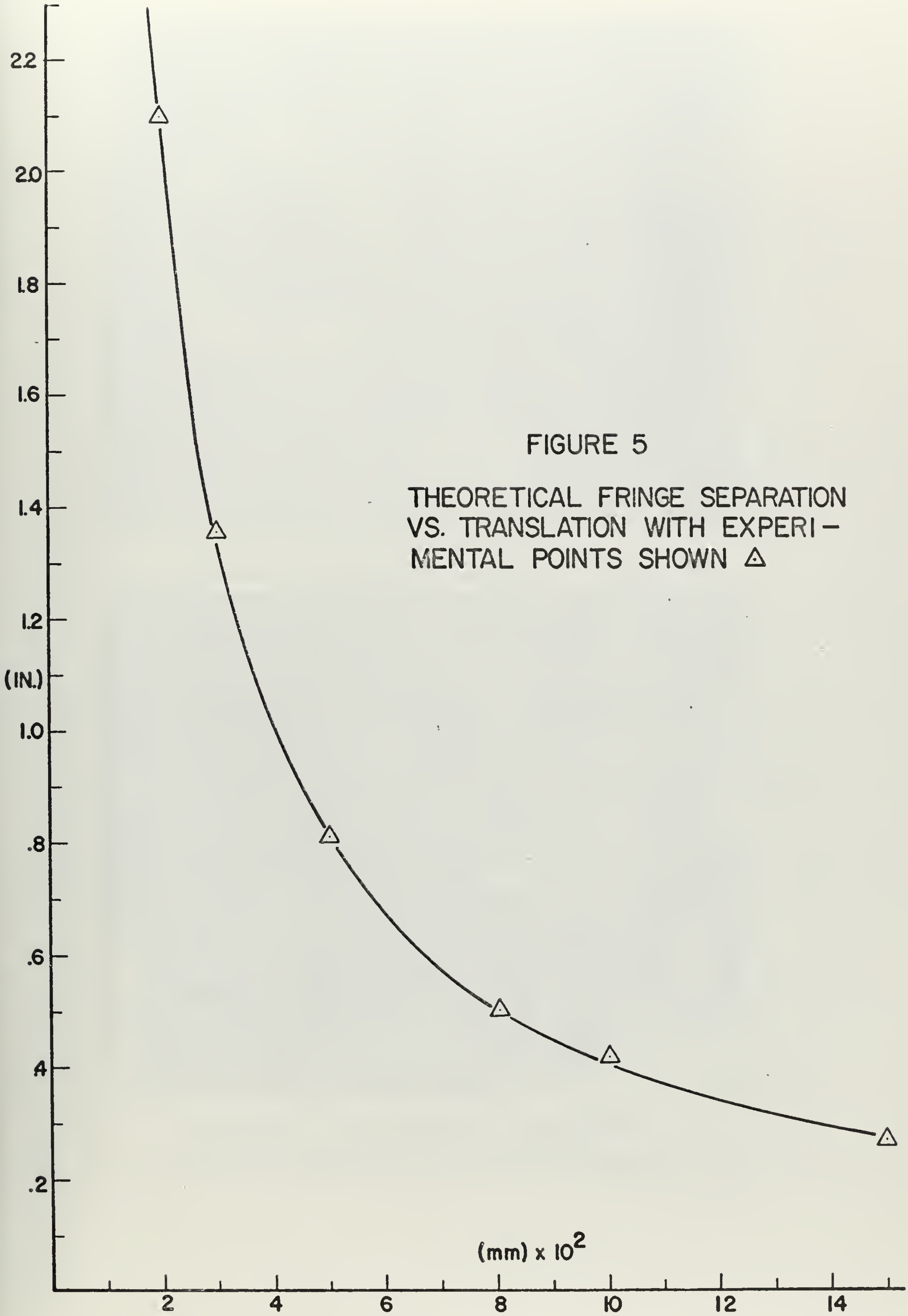
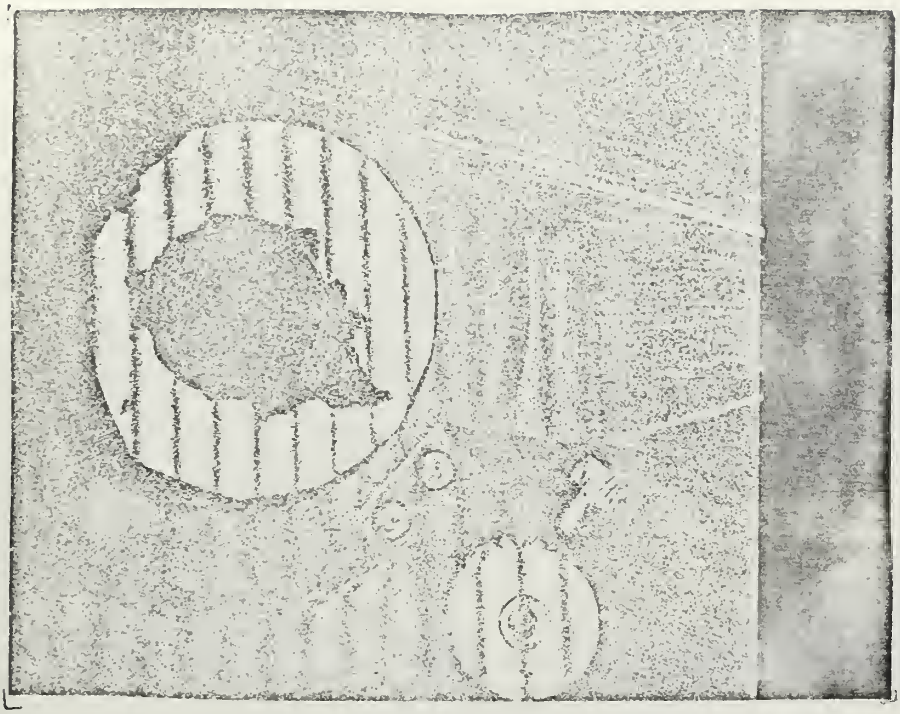
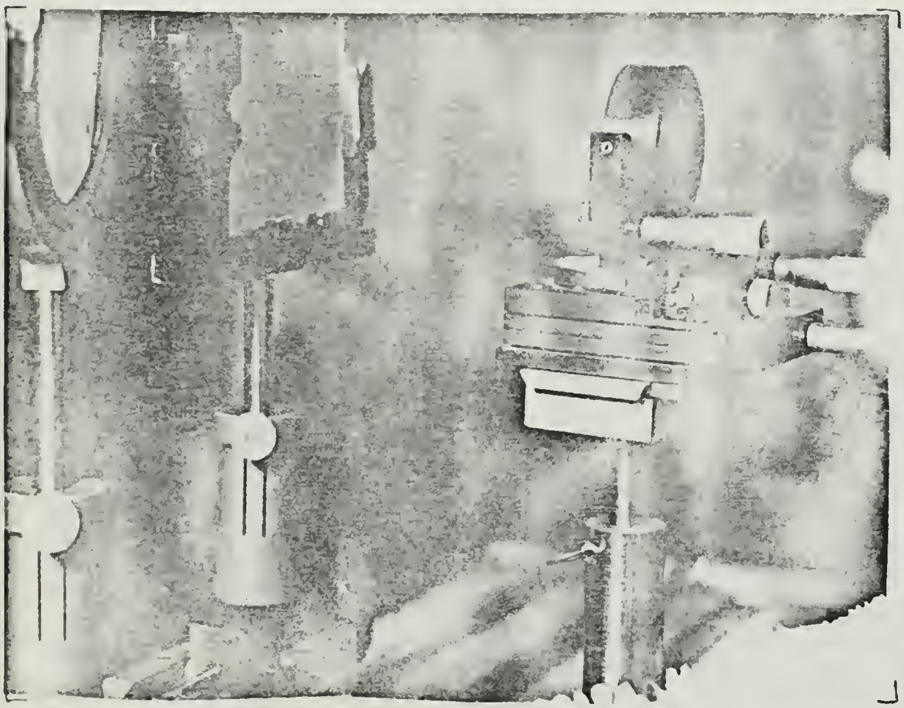


FIGURE 5

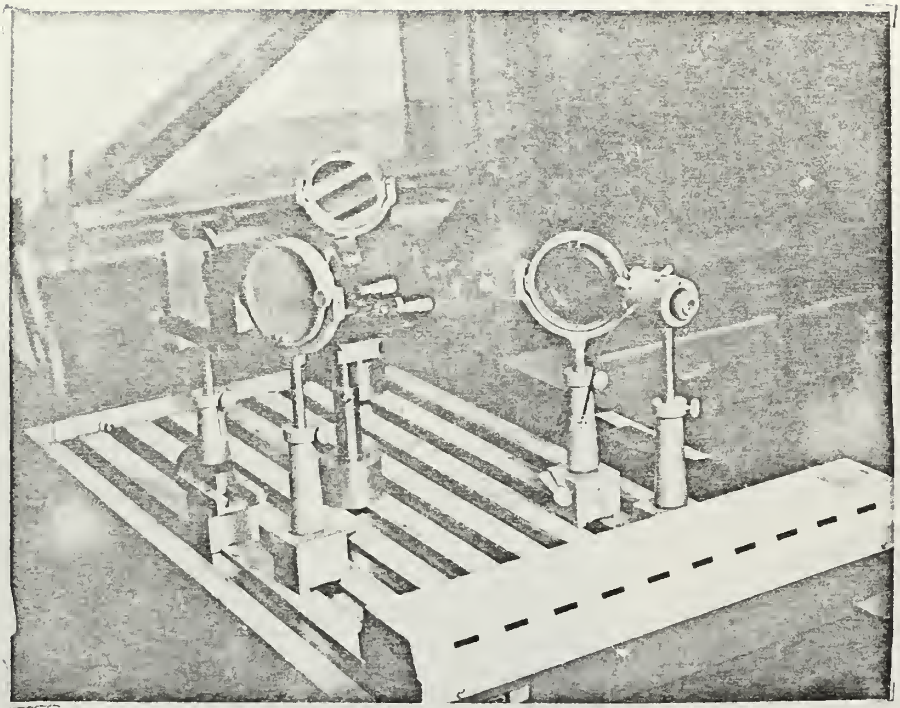
THEORETICAL FRINGE SEPARATION VS. TRANSLATION WITH EXPERIMENTAL POINTS SHOWN △



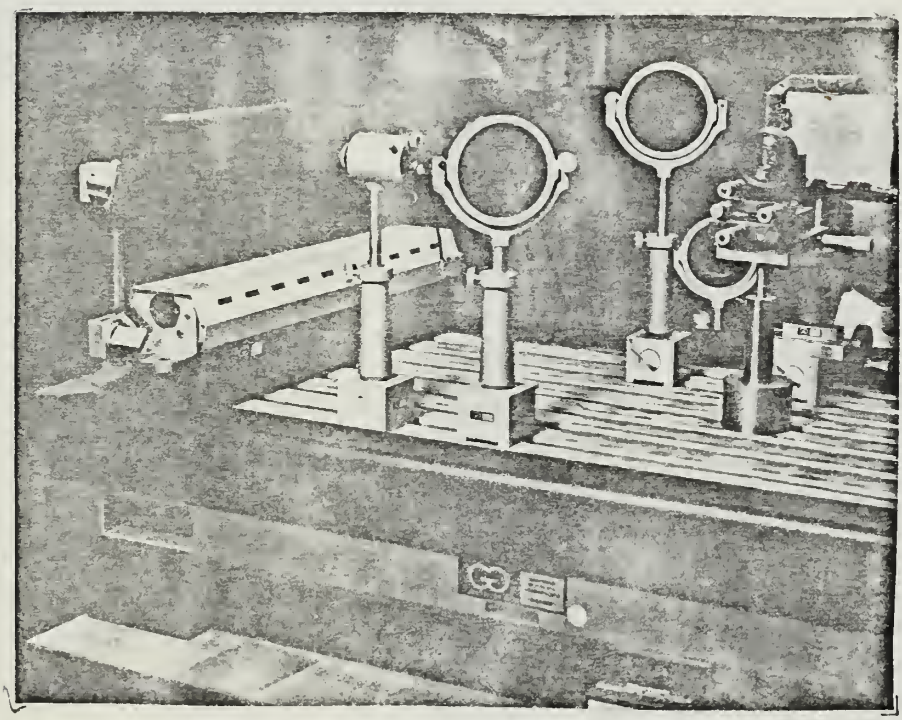
FRINGE PATTERN, ROTATION, 19.70 ARC-SECONDS



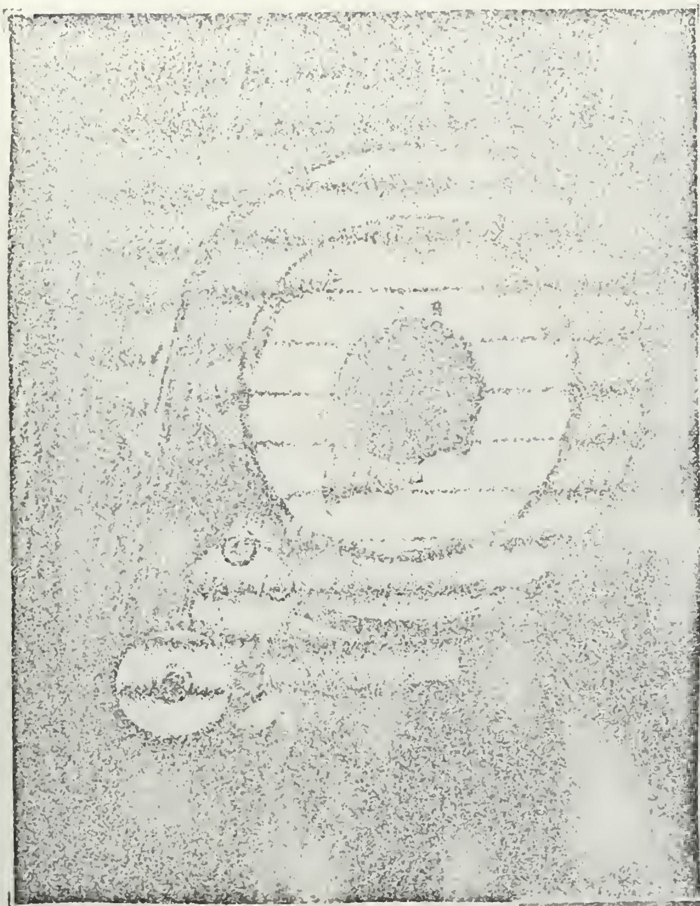
CLOSE-UP OF EXPERIMENTAL ARRANGEMENT
SHOWING DETAILS OF THE OPTICAL DEVICE



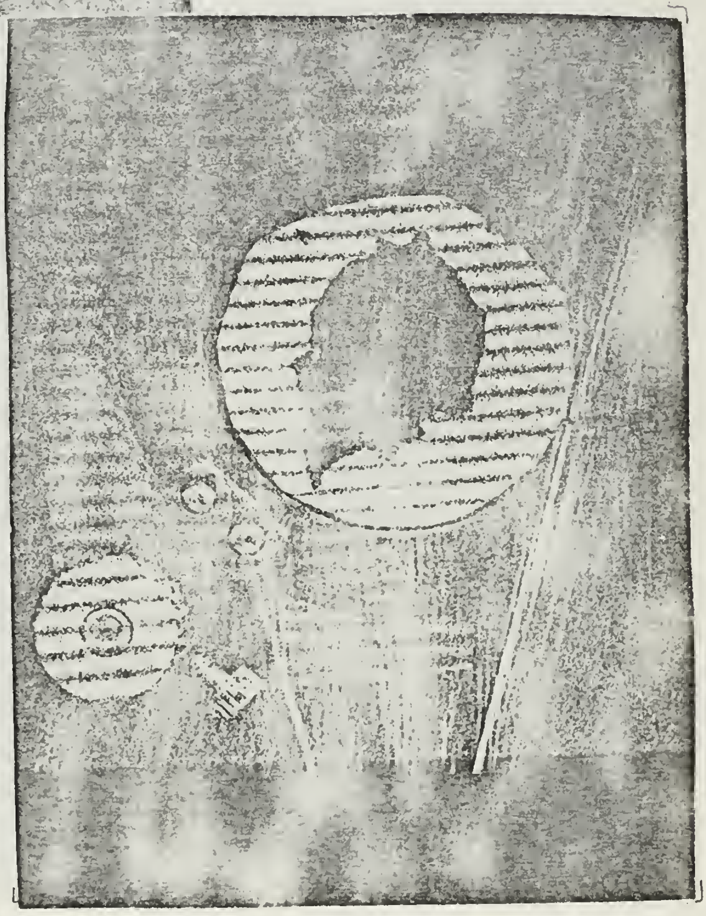
EXPERIMENTAL ARRANGEMENT OF OPTICAL TOOLS:
THE LASER IS ON THE FAR RIGHT.



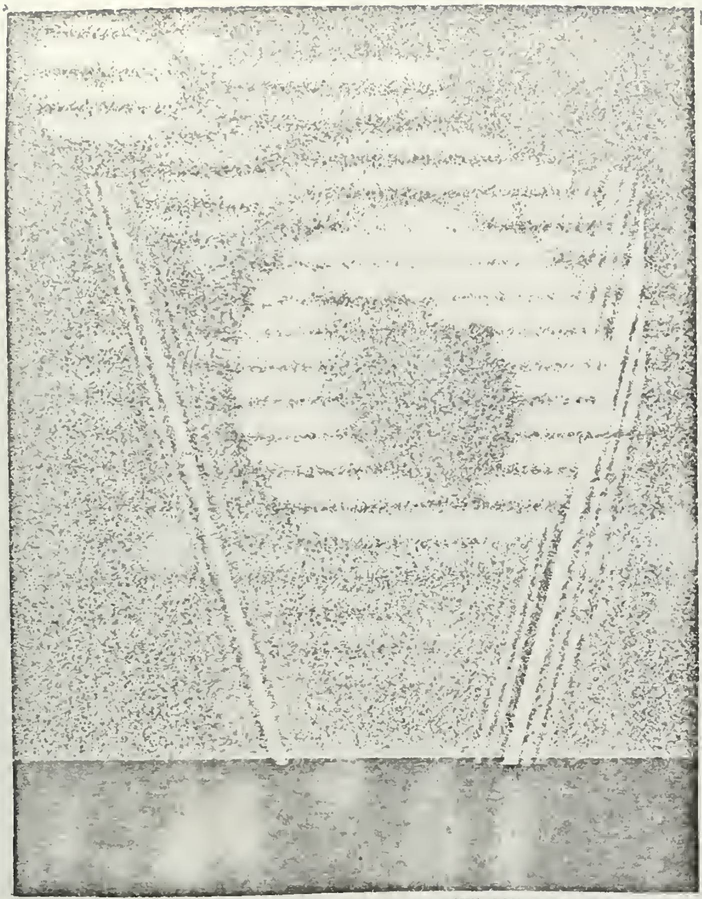
A SECOND VIEW OF THE EXPERIMENTAL SET-UP.
THE OPTICAL DEVICE AND PHOTOGRAPHIC PLATE
HOLDER ARE VISIBLE TO THE RIGHT.



FRINGE PATTERN
ROTATION
11.80 ARC-SECONDS



FRINGE PATTERN
ROTATION
27.60 ARC-SECONDS



FRINGE PATTERN
TRANSLATION
.02 MILLIMETER

FRINGE PATTERN
TRANSLATION
.03 MILLIMETER

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13. ABSTRACT

Analytic investigation was made of the light reflected by translated and rotated objects, and expressions were obtained for the interference patterns produced by this light. The technique of holography was used to experimentally measure fringe patterns produced by this interference, and to experimentally determine translation and rotation.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
olography						
terferometry						

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